LARGE NEUTRINO MIXING IN GRAND UNIFIED THEORIES

F. FERUGLIO
University of Padova and I.N.F.N., Padova, Italy
E-mail: feruglio@pd.infn.it

A non-minimal, semi-realistic version of supersymmetric SU(5) grand unified theory is discussed. The solution of the doublet-triplet splitting problem leads to a better agreement between the predicted and observed values of the low-energy strong coupling constant and to a prolongation of the proton lifetime. A $U(1)$ flavor symmetry allows to accommodate a realistic mass spectrum in the charged and in the neutral fermion sectors and protects doublet-triplet splitting and proton decay from dangerous radiative corrections or non-renormalizable operators.

Minimal versions of grand unified theories (GUTs) are plagued by severe fine-tuning problems.

- Doublet-triplet splitting – By far the most severe problem, it requires an unnatural adjustment of the superpotential parameters of one part in 10^{14}. Moreover, the tree-level solution to this problem can be spoiled either by radiative corrections when supersymmetry (SUSY) is broken, or by non renormalizable operators.
- Proton decay – Minimal SU(5) is ruled out by the recent SuperKamiokande data $\tau_p/BR(p \to K^+\bar{\nu}) > 2 \cdot 10^{33}$ ys (90% CL) [1]. Moreover generic, non-renormalizable operators contributing to proton decay and originating at the Planck scale M_{Pl} are expected to predict a proton lifetime of order 10^{20} ys.
- Wrong mass relations – The equality of down quark and charged lepton masses at the GUT scale in minimal SU(5) is inaccurate for the first two generations, even though it is correct order-of-magnitude wise. The whole neutrino sector is missing in minimal SU(5).
- Strong coupling constant – Gauge coupling unification in minimal GUT leads to a value of $\alpha_s(M_Z)$ that, although affected by large uncertainties, tends to be too large: $\alpha_s(M_Z) = 0.13 \pm 0.01$ for colour triplets at the GUT scale and supersymmetric particles close to 1 TeV.

There can also be additional issues more specific to the supersymmetric realization of GUT ideas, like for instance the supersymmetric flavour problem. In ref. [2] we discuss a semi-realistic model that addresses and solves the above problems in an extended version of SU(5), supplemented by a $\mathrm{U(1)_Q}$ flavour symmetry.

In this model the doublet triplet-splitting problem is solved by a variant of the missing partner mechanism employing, beyond 5 and $\bar{5}$, the 50, $\overline{50}$, $75 \equiv Y$

and $1 \equiv X$ SU(5) representations (see table 1) [3]. The multiplet Y, singlet under the flavour symmetry, breaks SU(5) down to SU(3) $\times$ SU(2) $\times$ U(1), while X, characterized by $Q = -1$, is the only field charged under $\mathrm{U(1)_Q}$ that acquires a large VEV. In the limit of exact SUSY, the doublets in 5 and $\bar{5}$ remain massless. The mass m_T of the effective triplet suppressing the dimension 5, $|\Delta B| = 1$ operators is proportional to $\langle Y \rangle^2 / \langle X \rangle$, where $\langle X \rangle$ is undetermined. Finally, operators like $5\bar{5}X^m Y^n$ $(m, n > 0)$, which potentially could destabilize the doublets, are forbidden by $\mathrm{U(1)_Q}$. When SUSY is broken, $\langle X \rangle$ acquires a VEV close to the cut-off Λ of the theory and a μ term can be generated à la Giudice-Masiero from a higher-dimensional term in the Kähler potential.

The spectrum of heavy particles associated to the missing partner mechanism produces two main effects.

- The strong coupling constant $\alpha_s(M_Z)$ receives large threshold corrections from the splitted Y supermultiplet. As a result, $\alpha_s(M_Z)$ is smaller than in minimal SU(5). Indeed values of m_T larger by a factor 20-30 than in minimal SU(5) are required to reconcile the prediction of $\alpha_s(M_Z)$ with the data, with a direct advantage for proton decay.

- The model is no longer asymptotically free, due to the large field content. The SU(5) coupling constant blows up at a scale $\overline{\Lambda}$ smaller than the Planck scale. We typically find $\Lambda \leq \overline{\Lambda} \approx 20\ M_{GUT}$.

Fermion masses are obtained from the $\mathrm{U(1)_Q}$ charge assignment given in table 1. As well known, abelian charges constrain the spectrum up to unknown coefficients of order one. It is possible to choose these coefficients in order to correctly reproduce quark masses, mixing angles and the CP violating phase. The model predicts $\tan\beta \approx O(1)$, which also moderates the proton decay amplitudes. The neutrino sector of the model is quite similar to the one discussed in ref. [4]. As a consequence of the $U(1)$ assignment and of the see-saw mechanism, a large mixing for atmospheric neutrinos is obtained. Such a mixing is directly related to a large mixing between the right-handed s and b quark fields, via the minimal SU(5) relation $m_e = {m_d}^T$, which is approximately valid also in the present model. This is the reason why a large mixing among leptons is compatible with small quark mixing angles, even in a GUT, where lepton and quarks belong to the same representations of the gauge group. The solar mixing angle is expected to be close to maximal and, numerically, the so called LOW and vacuum oscillation solutions are equally possible. Finally a θ_{13} angle of order 0.05 is predicted.

A well known obstacle in minimal SU(5) is the strict equality $m_e = {m_d}^T$, compatible with the third generation, but inexact for the first and the second families. The correction requires order-one adjustments that can be obtained in the present model by allowing, beyond the minimal $\Psi_{10} G_d \Psi_{\bar{5}} \bar{5}$ Yukawa coupling also the non-renormalizable term $1/\Lambda \Psi_{10} F_d \Psi_{\bar{5}} \bar{5} Y$. The Y multiplet

Table 1. Chiral Multiplets Quantum Numbers.

Field	SU(5)	$U(1)_Q$
H	5	-2
$\overline{H}$	$\overline{5}$	+1
H_{50}	50	2
$H_{\overline{50}}$	$\overline{50}$	-1
Y	75	0
X	1	-1
Ψ_{10}	10	(4,3,1)
$\Psi_{\bar{5}}$	$\bar{5}$	(4,2,2)
Ψ_1	1	(1,-1,0)

differentiates charged leptons from down quarks and we find:

$$m_d \approx \left[G_d + \frac{\langle Y\rangle}{\Lambda} F_d\right] \quad , \tag{1}$$

$$m_e^T \approx \left[G_d - 3\frac{\langle Y\rangle}{\Lambda} F_d\right] \;, \tag{2}$$

where the 3×3 matrices G_d and F_d are constrained by the flavour symmetry. It is interesting to observe that we can reproduce the relations $m_\tau \approx m_b$, $m_\mu \approx 3m_s$ and $m_e \approx m_d/3$, by taking $\langle Y\rangle/\Lambda$ of order 0.1, in agreement with $\Lambda \leq \overline{\Lambda} \approx 20\ M_{GUT}$. While the predictivity of the model is reduced because non-renormalizable operators are only suppressed by powers of M_{GUT}/Λ, still these corrections could explain the small distortion of the spectrum with respect to the minimal model.

Proton decay dominant amplitudes are derived from the dimension 5 superpotential:

$$w = \frac{1}{m_T}\left[Q\hat{A}QQ\hat{C}L + U^c\hat{B}E^cU^c\hat{D}D^c\right] \;, \tag{3}$$

which, although formally equal to that of the minimal model, exhibits four important differences:

- An effective triplet mass m_T, larger by a factor 20-30 than in minimal SU(5) leads to a suppression factor 400-900 in rate.

- An additional Yukawa coupling is allowed by the symmetries of the theory: $\Psi_{10} G_{\overline{50}} \Psi_{10} \overline{50}$. While the couplings of the conventional term $\Psi_{10} G_u \Psi_{10} 5$ are restricted by the up quark masses, the couplings of the new term are unconstrained, since $\langle \overline{50} \rangle = 0$. We obtain:

$$\hat{B} = -2\hat{A} = \left[G_u - \frac{c_2 \langle Y \rangle}{c_4 \langle X \rangle} G_{\overline{50}} \right] , \tag{4}$$

  where c_2 and c_4 are dimensionless coefficients. As a consequence, a large region in parameter space exists where a sizeable destructive interference between the G_u and the $G_{\overline{50}}$ contributions can occur.

- Also the $\hat{C}$ and $\hat{D}$ couplings are distorted:

$$\hat{C} = \left[-G_d - \frac{\langle Y \rangle}{\Lambda} F_d \right] , \tag{5}$$

$$\hat{D} = \left[G_d - \frac{\langle Y \rangle}{\Lambda} F_d \right] . \tag{6}$$

  This modification, however, has a not-too-large effect on proton decay rates.

- The non-renormalizable operators that could originate at the cut-off scale Λ are controlled by the flavour symmetry and lead to a contribution to the proton decay amplitude that can be comparable to the one coming from the color triplet exchange.

As a result, the predicted range for the proton decay rates considerably extends with respects to that of minimal SU(5), allowing values that are not incompatible with the present limits and are testable in the next generation of experiments. In particular, our numerical estimate gives $8 \cdot 10^{31}$ ys $< \tau_p / BR(p \to K^+ \bar{\nu}) < 3 \cdot 10^{34}$ ys and $2 \cdot 10^{32}$ ys $< \tau_p / BR(p \to \pi^+ \bar{\nu}) < 8 \cdot 10^{34}$ ys.

In summary, it is a remarkable feature of the model that the presence of the representations 50, $\overline{50}$ and 75, demanded by the missing partner mechanism for the solution of the doublet-triplet splitting problem, directly produces, through threshold corrections at M_{GUT}, a decrease of the value of $\alpha_s(m_Z)$ that corresponds to coupling unification and an increase in the effective mass that mediates proton decay. As a consequence the value of the strong coupling is in better agreement with the experimental value and the proton decay rate is smaller by a factor 400-900 than in the minimal model. The presence of these large representations also has the consequence that the asymptotic freedom of SU(5) is spoiled and the associated gauge coupling becomes non perturbative below M_{Pl}. We argue that this property far from being unacceptable can actually play an important role to obtain better results for fermion masses.

Acknowledgments

I warmly thank Guido Altarelli and Isabella Masina for the enjoyable collaboration on which this talk is based. The present work has been partially supported by the European Program HPRN-CT-2000-00148 (network Across The Energy Frontier).

References

1. Masato Shiozawa, "*Mini-Review on Proton Decay Searches*", presented at XXXth International Conference on High Energy Physics (ICHEP2000), Osaka, July 27-August 2, 2000.
2. G. Altarelli, F. Feruglio and I. Masina, JHEP **0011** (2000) 040 [hep-ph/0007254].
3. Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B396 (1997) 150 [hep-ph/9611277].
4. G. Altarelli and F. Feruglio, JHEP **9811** (1998) 021 [hep-ph/9809596]; Phys. Lett. B **451** (1999) 388 [hep-ph/9812475].